DATE : 02/09/2020

Time : 3 hrs. Max. Marks : 300

Answers & Solutions

for

SEPT - JEE - 2020

Lakshya

Forum for Competitions™

PART : PHYSICS

SECTION – 1 : (Maximum Marks : 80)

Straight Objective Type

This section contains **20 multiple choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. A particle of mass m with an initial velocity $u\hat{i}$ collides perfectly elastically with a mass 3 m at rest. It moves with a velocity $v\hat{j}$ after collision, then, v is given by

(1) $v = \sqrt{\frac{2}{3}}u$ (2) $v = \frac{u}{\sqrt{3}}$ (3) $v = \frac{1}{\sqrt{6}}u$ (D) $v = \frac{u}{\sqrt{2}}$

Ans. **(4)**

Sol. From momentum conservation

$$mu\hat{i} + 0 = mv\hat{j} + 3m\vec{v}'$$

$$\vec{v}' = \frac{u}{3}\hat{i} - \frac{v}{3}\hat{j}$$

From kinetic energy conservation $\frac{1}{2}mu^2 = \frac{1}{2}mv^2 + \frac{1}{2}(3m)\left(\left(\frac{u}{3}\right)^2 + \left(\frac{v}{3}\right)^2\right)$

Solving $v = \frac{u}{\sqrt{2}}$

2. Two identical strings X and Z made of same material have tension T_x and T_Z in then If their fundamental frequencies are 450 Hz and 300 Hz, respectively, then the ratio T_x/T_Z is :

(1) 1.25 (2) 0.44 (3) 1.4 (4) 2.25

Ans. **(4)**

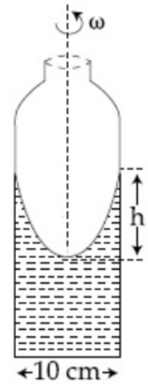
Sol. $f_x = \frac{1}{2\ell}\sqrt{\frac{T_x}{\mu}}$

$$f_y = \frac{1}{2\ell}\sqrt{\frac{T_y}{\mu}}$$

$$\frac{f_x}{f_y} = \frac{450}{300} = \sqrt{\frac{T_x}{T_y}}$$

$\Rightarrow T_x/T_y = 9/4 = 2.25$

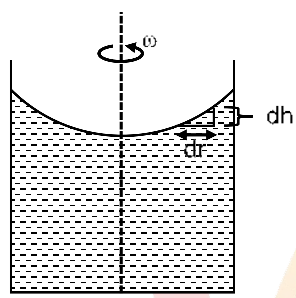
3. A cylindrical vessel containing a liquid is rotated about its axis so that the liquid rises at its sides as shown in the figure. The radius of vessel is 5 cm and the angular speed of rotation is ω rad s^{-1}. The difference in the height, h (in cm) of liquid at the centre of vessel and at the will be :



(1) $\frac{5\omega^2}{2g}$ (2) $\frac{2\omega^2}{25g}$ (3) $\frac{25\omega^2}{2g}$ (4) $\frac{2\omega^2}{5g}$

Ans. **(3)**

Sol.



$\rho dr\omega^2 r = \rho g dh$

$\omega^2 \int_0^R r dr = g\int_0^h dh$

$\frac{\omega^2 R^2}{2} = gh$

$h = \frac{\omega^2 R^2}{2g} = \frac{25\omega^2}{2g}$

4. The least count of the main scale of a vernier callipers is 1 mm. Its vernier scale is divided into 10 divisions and coincide with 9 divisions of the main scale. When jaws are touching each other, the 7^{th} division of vernier scale coincides with a division of main scale and the zero of vernier scale is lying right side of the zero of main scale. When this vernier is used to measure length of cylinder the zero of the vernier scale between 3.1 cm and 3.2 cm and 4^{th} VSD coincides with a main scale division. The length of the cylinder is (VSD is vernier scale division)

(1) 2.99 cm (2) 3.07 cm (3) 3.21 cm (4) 3.2 cm

Ans. **(2)**

Sol. Zero Error = 0 + 7 × 0.1 = 0.070

Vernier reading = (3.1 + 4 × 0.01) –0.07 = 3.07

to be ideal and the oxygen bond to be rigid, the total internal energy (in units of RT) of the mixture is :

(1) 15 (2) 13 (3) 20 (4) 11

Ans. **(1)**

Sol. $\frac{f_1n_1RT_1}{2}+\frac{f_2n_2RT_2}{2}=3\times\frac{5}{2}RT+\frac{5}{2}\times 3RT=15$

6. Interference fringes are observed on a screen by illuminating two thin slits 1 mm apart with a light source (λ = 632.8 nm). The distance between the screen and the slits is 100 cm. If a bright fringe is observed on a screen at distance of 1.27 mm from the central bright fringe, then the path difference between the waves, which are reaching this point from the slits is close to :

(1) 2.87 (2) 2 nm (3) 1.27 μm (4) 2.05 μm

Ans. **(3)**

Sol. $\Delta P = d\sin\theta$

$= d\theta$

$= \frac{dy}{D} = \frac{10^{-3}\times 1.270mm}{1m} = 1.27\ \mu m$

7. An amplitude modulated waves is represented by expression v_m = 5 (1 + 0.6 cos 6280t) sin(211 × 10^4t) volts. The minimum and maximum amplitudes of the amplitude modulated wave are, respectively:

(1) 5V, 8V (2) $\frac{5}{2}$V , 8V (3) 3V, 5V (4) $\frac{3}{2}$V , 5V

Ans. **(2)**

Sol. From Given Equation

$\mu = 0.6$

$A_m = \mu A_c$

$\frac{A_{max.}+A_{min}}{2}=A_c=5$ ......(1)

$\frac{A_{max.}-A_{min}}{2}=3$ ......(2)

From Equation (1) + (2)

$A_{max} = 8$

From Equation (1) – (2)

$A_{min} = 2$

8. The mass density of a spherical galaxy varies as $\frac{K}{r}$ over a large distance 'r' from its center. In that region, a small star is in a circular orbit of radius R. Then the period of revolution, T depends on R as :

(1) $T^2\alpha\frac{1}{R^3}$ (2) $T^2 \alpha R$ (3) $T \alpha R$ (4) $T^2 \alpha R^3$

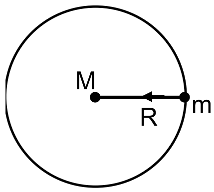
Ans. **(2)**

Sol. $M=\int\rho dV$

$M=\int_0^{r=R_0}\frac{k}{r}4\pi r^2dr$

$$M = 4\pi k\int_0^{R_0} r\,dr$$

$$M = \frac{4\pi kR_0^2}{2} = 2\pi kR^2$$



$$F_G = \frac{GMm}{R_0^2} = m\omega_0^2 R$$

$$\Rightarrow \quad \frac{G\frac{4\pi kR^2}{2}}{R^2} = \omega_0^2 R \quad \Rightarrow \quad \omega_0 = \sqrt{\frac{2\pi KG}{R}}$$

$$\because\ T = \frac{2\pi}{\omega_0} = \frac{2\pi\sqrt{R}}{\sqrt{2\pi KG}} = \sqrt{\frac{2\pi R}{KG}}$$

$$\Rightarrow T^2 \propto R$$

9. A beam of protons with speed 4×10^5 ms^{-1} enters a uniform magnetic field of 0.3T at an angle of 60º to the magnetic field. the pitch of the resulting helical path of protons is close to : (Mass of the proton = 1.67×10^{-27} kg, charge of the proton = 1.69×10^{-19} C)

(1) 12 cm (2) 2 cm (3) 4 cm (4) 5 cm

Ans. **(3)**

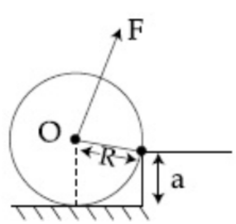
Sol. Pitch $= (V\cos\theta)T$

$$= (V\cos\theta)\frac{2\pi m}{eB}$$

$$= (4 \times 10^5 \cos 60°)\frac{2\pi}{0.3\times 10}\left(\frac{1.67\times 10^{-27}}{1.69\times 10^{19}}\right)$$

$$= 4 \text{ cm}$$

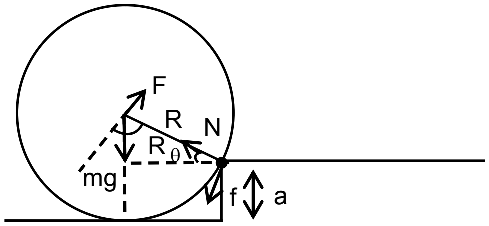
10. A uniform cylinder of mass M and radius R is to be pulled over a step of height a (a < R) by applying a force F at its centre 'O' perpendicular to the plane through the axes of the cylinder on the edge of the step (see figure). The minimum value of F required is :



(1) $Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$ (2) $Mg\sqrt{1-\frac{a^2}{R^2}}$ (3) $Mg\sqrt{\left(\frac{R}{R-a}\right)-1}$ (4) $Mg\frac{a}{R}$

Ans. **(1)**

Sol.

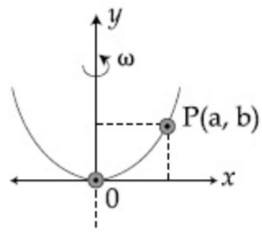


FR > mg cosθR

F > mgcosθ

$$F > mg\frac{\sqrt{R^2-(R-a)^2}}{R} \Rightarrow Mg\sqrt{1-\left(\frac{R-a}{R}\right)^2}$$

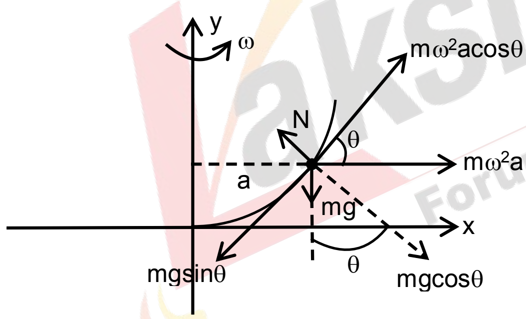
11. A bead of mass m stays at point P (a, b) on a wire bent in the shape of a parabola y = Cx^2 and rotating with angular speed ω (see figure). The value of ω is (neglect friction)



(1) $\sqrt{2gC}$ (2) $\sqrt{\frac{2gC}{ab}}$ (3) $\sqrt{\frac{2g}{C}}$ (4) $2\sqrt{2gC}$

Ans. **(4)**

Sol.



$m\omega^2 a\cos\theta = mg\sin\theta$

$\omega = \sqrt{\frac{g\tan\theta}{a}}$

$y = 4cx^2$

$\tan\theta = \frac{dy}{dx} = 8xC$

$(\tan\theta)_{a,b} = 8aC$

$\omega = \sqrt{\frac{g\times 8ac}{a}} = 2\sqrt{2gc}$

12. If speed V, area A and force F are chosen as fundamental units, then the dimension of Young's modulus will be :

(1) FA^2V^{-3} (2) FA^2V^{-1} (3) FA^2V^{-2} (4) $FA^{-1}V^0$

Ans. **(4)**

Sol. $Y \propto F^a V^b A^c$ $\quad Y = \left(\frac{F}{A}\right)$

$$\frac{MLT^{-2}}{L^2} \propto (M^1L^1T^{-2})^a (L^1T^{-1})^b (L^2)^c$$

$M^1L^{-1}T^{-2} \propto M^a L^{a+b+2c} T^{-2a-b}$

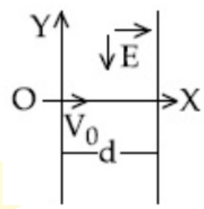
$a + b + 2c = -1$

$-2a + b = -2$

$a = 1, b = 0, c = -1$

$Y = F^1v^0A^{-1}$

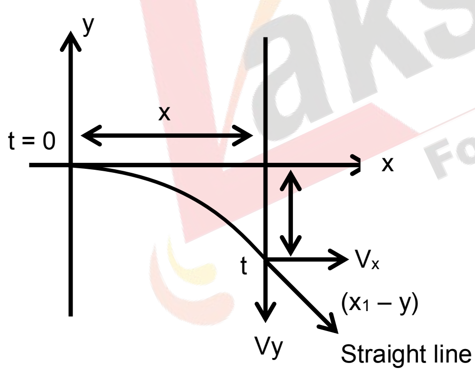
13. A charged particle (mass m and charge q) moves along X axis with velocity V_0. When it passes through the origin it enters a region having uniform electric field $\vec{E} = -E\hat{j}$ which extends upto x = d. Equation of path of electron in the region x > d is :



(1) $y = \frac{qEd}{mV_0^2}x$ (2) $y = \frac{qEd}{mV_0^2}(x-d)$ (3) $y = \frac{qEd}{mV_0^2}\left(\frac{d}{2}-x\right)$ (4) $y = \frac{qEd^2}{mV_0^2}x$

Ans. **(3)**

Sol.



x > d path is straight line

$$\frac{-y = \frac{1}{2}at^2}{x-d} = \frac{at}{V_0}$$

$$\frac{-y - \frac{1}{2}at^2}{at} = \frac{x-d}{V_0}$$

$$\frac{-y}{at} - \frac{1}{2}\frac{d}{V_0} = \frac{x}{V_0} - \frac{d}{V_0}$$

$$\frac{-myV_0}{qEd} = \frac{x}{V_0} - \frac{d}{2V_0}$$

$$y = \frac{-qEd}{mV_0}\left(\frac{x}{V_0} - \frac{d}{2V_0}\right)$$

$$y- = \frac{qEd}{mV_0^2}\left(\frac{d}{2} - x\right)$$

14. In a reactor, 2 kg of $_{92}U^{235}$ fuel is fully used up in 30 days. The energy released per fission is 200 Mev. given that the Avogadro number, N = 6.023 × 10^{26} per kilo mole and 1 eV = 1.6 × 10^{-19} J. The power output of the reactor is close to :

(1) 54 MW (2) 60 MW (3) 125 MW (4) 35 MW

Ans. **(2)**

Sol. $P = \frac{E}{t}$

$$= \frac{2}{235} \times \frac{6.023 \times 10^{26} \times 200 \times 1.6 \times 10^{-19}}{30 \times 24 \times 60 \times 60} = 60W$$

15. A plane electromagnetic wave, has frequency of 2.0 × 10^{10} Hz and its energy density is 1.02 × 10^{-8} J/m^3 in vacuum. The amplitude of the magnetic field of the wave is close to ($\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$ and speed of light = 3 × 10^8 ms^{-1}) :

(1) 160 nT (2) 180 nT (3) 190 nT (4) 150 nT

Ans. **(1)**

Sol. Energy Density = $\frac{1}{2}\frac{B^2}{\mu_0}$

$$B = \sqrt{2 \times \mu_0 \times \text{Energy density}}$$

$$B = \sqrt{2 \times 4\pi \times 10^{-7} \times 1.02 \times 10^{-8}} = 160 \times 10^{-9} = 160 \text{ nT}$$

16. Train A and train B are running on parallel tracks in the opposite directions with speed of 36 km/hour and 72 km/hour, respectively. A person is walking in train A in the direction opposite to its motion with a speed of 1.8 km/hour. Speed (in ms^{-1}) of this person as observed from train B will be close to : (take the distance between the tracks as negligible)

(1) 30.5 ms^{-1} (2) 29.5 ms^{-1} (3) 31.5 ms^{-1} (4) 28.5 ms^{-1}

Ans. **(2)**

Sol.

V_A = 36 km/hr = 10 m/s
V_B = –72 km/hr = –20 m/s

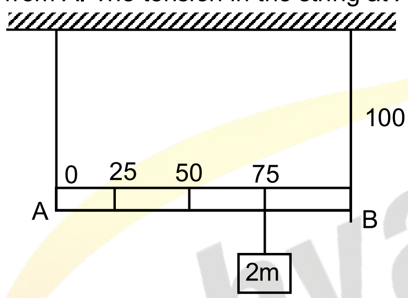
$V_{MA} = -1.8$ km/hr $= -0.5$ m/s
$V_{man,\,B} = V_{man,\,A} + V_{A,\,B}$
$= V_{man,\,A} + V_A - V_B$
$= -0.5 + 10 - (-20)$
$= -0.5 + 30$
$= 29.5$ m/s

17. Magnetic materials used for making permanent magnets (P) and magnets in a transformer (T) have different properties of the following, which property best matches for the type of magnet required ?
(1) T : Large retentivity, large coercivity
(2) P : Large retentivity, large coercivity
(3) T : Large retentivity, small coercivity
(4) T : Small retentivity, large coercivity

Ans. **(2)**

Sol. **Based on theory**

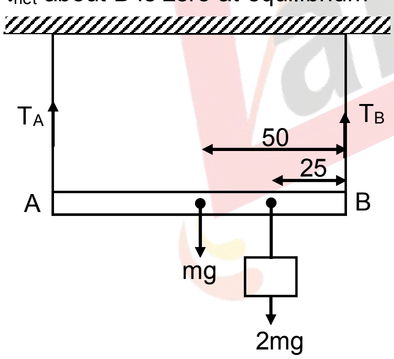
18. Shown in the figure is rigid and uniform one meter long rod AB held in horizontal position by two strings tied to its ends and attached to the ceiling. The rod is of mass 'm' and has another weight of mass 2m hung at a distance of 75 cm from A. The tension in the string at A is :



(1) 0.75 mg (2) 1 mg (3) 0.5 mg (4) 2mg

Ans. **(2)**

Sol. τ_{net} about B is zero at equilibrium



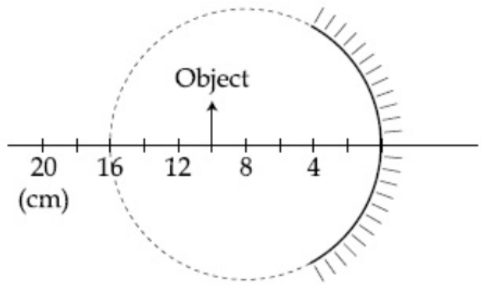
$T_A\,100 - mg \times 50 - 2\,mg \times 25 = 0$
$T_A\,100 = 100\,mg$
$T_A = 1\,mg$

19. Consider four conducting materials copper, tungsten, mercury and aluminum with resistivity ρ_C, ρ_T, ρ_M and ρ_A respectively. Then :
(1) $\rho_C > \rho_A > \rho_T$ (2) $\rho_A > \rho_T > \rho_C$ (3) $\rho_A > \rho_M > \rho_C$ (4) $\rho_M > \rho_A > \rho_C$

Ans. **(4)**

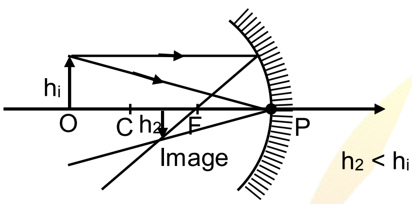
Sol. $\rho_m = 98 \times 10^{-8}$
$\rho_A = 2.65 \times 10^{-8}$
$\rho_C = 1.724 \times 10^{-8}$
$\rho_T = 5.65 \times 10^{-8}$

20. A spherical mirror is obtained as shown in the figure from a hollow glass sphere. if an object is positioned infront of the mirror, what will be the nature and magnification of the image of the object? (Figure drawn as schematic and not to scale)



(1) Inverted, real and magnified
(2) Erect, virtual and unmagnified
(3) Inverted, real and unmagnified
(4) Erect, virtual and magnified

Ans. **(3)**
Sol.



SECTION – 2 : (Maximum Marks : 20)

- This section contains **FIVE (05)** questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.
- If the numerical value has more than two decimal places **truncate/round-off** the value upto **TWO** decimal places.
  - Full Marks : **+4** If ONLY the correct option is chosen.
  - Zero Marks : **0** In all other cases

21. A 5μF capacitor is charged fully by a 220 V supply. It is then disconnected from the supply and is connected in series to another uncharged 2.5 μF capacitor. If the energy change during the charge redistribution is $\frac{X}{100}$ J then value of X to the nearest integer is :

Ans. **4**

Sol. $C_1 = 5\mu F$ $V_1 = 220$ Volt

$C_2 = 2.5\ \mu F$ $V_2 = 0$

Heat loss; $\Delta H = U_i - U_f = \frac{1}{2}\frac{c_1c_2}{c_1+c_2}(v_1 - v_2)^2$

$= \frac{1}{2} \times \frac{5 \times 2.5}{(5+2.5)}(220-0)^2 \mu J$

$= \frac{5}{2 \times 3} \times 22 \times 22 \times 100 \times 10^{-6} J$

$= \frac{5\times11\times22}{3}\times10^{-4}J = \frac{55\times22}{3}\times 10^{-4}$ J

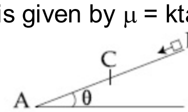
$= \frac{1210}{3}\times 10^{-4}$ J $= \frac{1210}{3}\times 10^{-3}$J $= 4\times 10^{-2}$

According to questions

$\frac{x}{100} = 4\times10^{-2}$

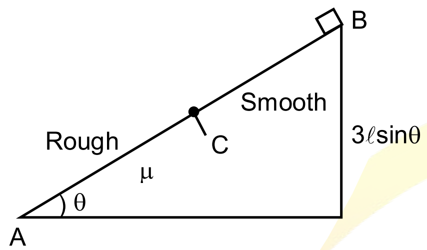
So, x = 4

22. A small block starts slipping down from a point B on an inclined plane AB, which is making an angle θ with the horizontal section BC is smooth and the remaining section CA is rough with a coefficient of friction μ. It is found that the block comes to rest as it reaches the bottom (point A) of the inclined plane. If BC = 2AC, the coefficient of friction is given by $\mu = k\tan\theta$. The value of k is ……….



Ans. 3

Sol. Let AC = ℓ ∴ BC = 2ℓ ∴ AB = 3ℓ



Apply work – Energy theorem

$W_f + W_{mg} = \Delta KE$

$mg(3\ell)\sin\theta - \mu mg\cos\theta(\ell) = 0 + 0$

$\mu mg\cos\theta\ell = 3mg\ell\sin\theta$

$\mu = 3\tan\theta = k\tan\theta$

∴ k = 3

23. An engine takes in 5 moles of air at 20°C and 1 atm, and compresses it adiabatically to 1/10th of the original volume. Assuming air to be a diatomic ideal gas made up of rigid molecules, the change in its internal energy during this process comes out to be XkJ. The value of X to the nearest integer is :

Ans. 46

Sol. $T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$

$T_2 = T_1\left(\frac{V_1}{V_2}\right)^{\gamma-1}$

$= T_1(10)^{\frac{7}{5}-1}$

$T_2 = T_1(10)^{2/5}$

$\Delta V = \frac{5}{2}nR;\ \frac{5}{2}\times5\times3[10^{2/5}-1](293)$

$\frac{625}{6}\times1.5\times293 = 461440$

≈ 46ks

24. When radiation of wavelength A is used to illuminate a metallic surface, the stopping potential is V. When the same surface is illuminated with radiation of wavelength 3A, the stopping potential is $\frac{V}{4}$. If the threshold wavelength for the metallic surface is nλ then value of n will be :

Ans. **9**

Sol. $\frac{hc}{\lambda} = \phi + eV$ ...(1)

$\frac{hc}{3\lambda} = \phi + \frac{eV}{4}$ ....(2)

from (1) & (2)

$\frac{hc}{\lambda}\left(1-\frac{1}{3}\right) = \frac{3}{4}eV$

$\frac{hc}{\lambda}\frac{2}{3} = \frac{3}{4}eV$

$eV = \frac{8}{9}\frac{hc}{\lambda}$

$\frac{hc}{\lambda} = \phi + \frac{8}{9}\frac{hc}{\lambda}$

$\phi = \frac{hc}{9\lambda} = \frac{hc}{\lambda_{th}}$

$\lambda_{th} = 9\lambda$

$\therefore$ k = 9

25. A circular coil of radius 10 cm is placed in a uniform magnetic field of 3.0×10^{-5} T with its plane perpendicular to the field initially. It is rotated at constant angular speed about an axis along the diameter of coil and perpendicular to magnetic field so that it undergoes half of rotation in 0.2 s. The maximum value of EMF induced (in μV) in the coil will be close to the integer…

Ans. **15**

Sol. Flux as a function of time $\phi = \vec{B}.\vec{A} = AB\cos(\omega t)$

Emf induced,

$e = \frac{-d\phi}{dt} = AB\omega\sin(\omega t)$

Max. value of Emf = $AB\omega$

= $\pi R^2 B\omega$

= $3.14 \times 0.1 \times 0.1 \times 3 \times 10^{-5} \times \frac{\pi}{0.2}$ = 15

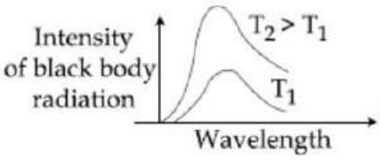
PART : CHEMISTRY

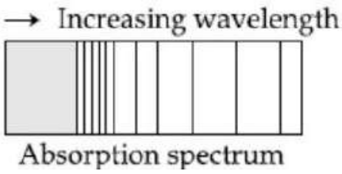
SECTION – 1 : (Maximum Marks : 80)

Straight Objective Type

This section contains **20 multiple choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. The figure that is not a direct manifestation of the quantum nature of atom is :

(1)


(2)


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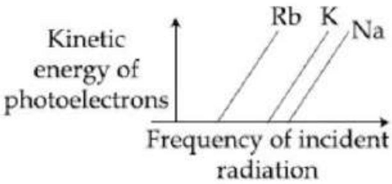
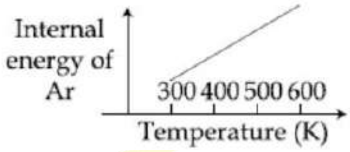
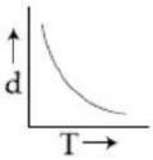


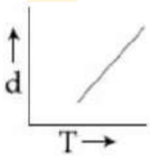
(4)


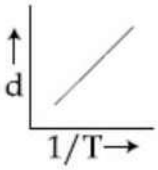
Ans. (4)

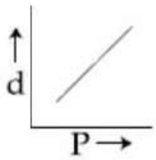
Sol. 1, 2 and 3 are according to quantum theory but (4) is statement of kinetic theory of gases.

2. Which one of the following graphs is not correct for ideal gas ?

(1)


(2)


(3)


(4)


d = Density, P = Pressure, T = Temperature

Ans. **(4)**

Sol^n. For ideal gas

PM = dRT

$$d = \left[\frac{PM}{R}\right]\frac{1}{T}$$

So graph between d Vs T is not straight line.

3. For the following Assertion and Reason, the correct option is

Assertion (A) : When Cu (II) and sulphide ions are mixed, they react together extremely quickly to give a solid.

Reason (R) : The equilibrium constant of Cu^{2+} (aq) + S^{2-} (aq) $\rightleftharpoons$ CuS(s) is high because the solubility product is low.

(1) (A) is false and (R) is true.

(2) Both (A) and (R) are false.

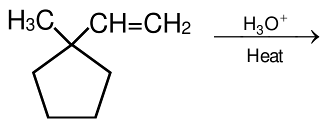
(3) Both (A) and (R) are true but (R) is not the explanation for (A).

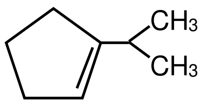
(4) Both (A) and (R) are true and (R) is the explanation for (A).

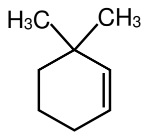
Ans. (3)

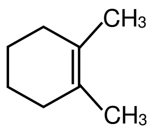
Sol. Rate of chemical reaction has nothing to do with value of equilibrium constant.

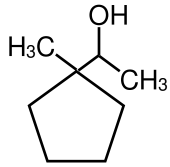
4. The major product in the following reaction is :



(1) 

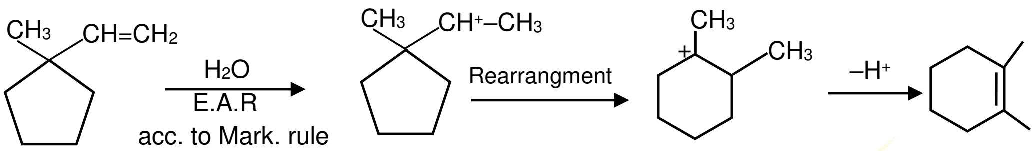
(2) 

(3) 

(4) 

Ans. **(3)**

Sol.



5. While titration dilute HCl solution with aqueous NaOH, which of the following will not be required ?

(1) Clamp and phenolphthalein
(2) Burette and porcelain tile
(3) Bunsen burner and measuring cyclinder
(4) Pipette and distilled water

Ans. (3)

Sol. In this acid base Titrating there is no use of Bunsen burner and measuring cylinder other laboratory equipments will be required for getting the end point of titration.

6. On heating compound (A) gives a gas (B) which is a constituent of air. This gas when treated with H_2 in the presence of a catalyst gives another gas (C) which is basic in nature. (A) should not be :

(1) $(NH_4)_2Cr_2O_7$
(2) NH_4NO_2
(3) $Pb(NO_3)_2$
(4) NaN_3

Ans. **(3)**

Sol. (1) $Pb(NO_3)_2 \xrightarrow{\Delta} PbO + 2NO_2$

(2) $NH_4NO_2 \xrightarrow{\Delta} N_2 + 2H_2O$

(3) $(NH_4)_2Cr_2O_7 \xrightarrow{\Delta} N_2 + Cr_2O_3 + H_2O$

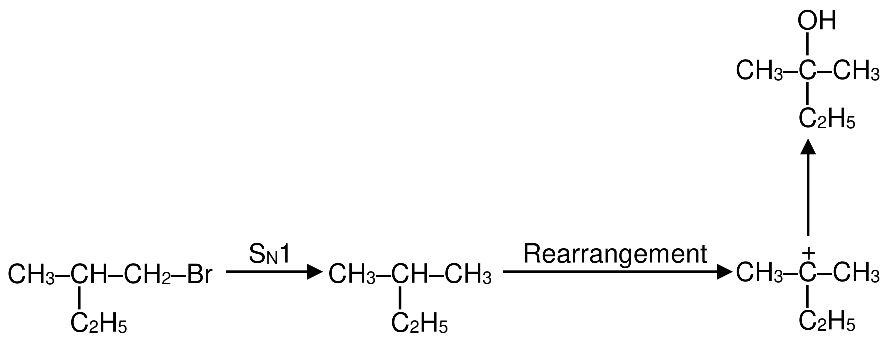
(4) $NaN_3 \xrightarrow{300^\circ C} 3N_2 + 2Na$

7. Which of the following compounds will show retention in configuration on nucleophile substitution by OH^- ion ?

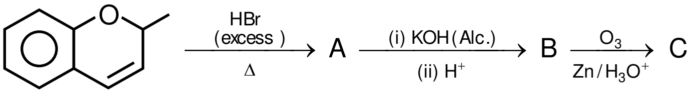
(1) $CH_3-CH(C_2H_5)-CH_2Br$
(2) $CH_3-CH(CH_3)-Br$
(3) $CH_3-C(Br)(C_6H_{13})-H$
(4) $CH_3-CH(C_6H_5)-Br$

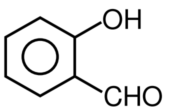
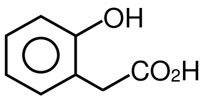
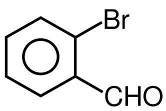
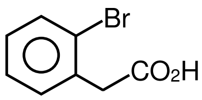
Ans. **(1)**

Sol.



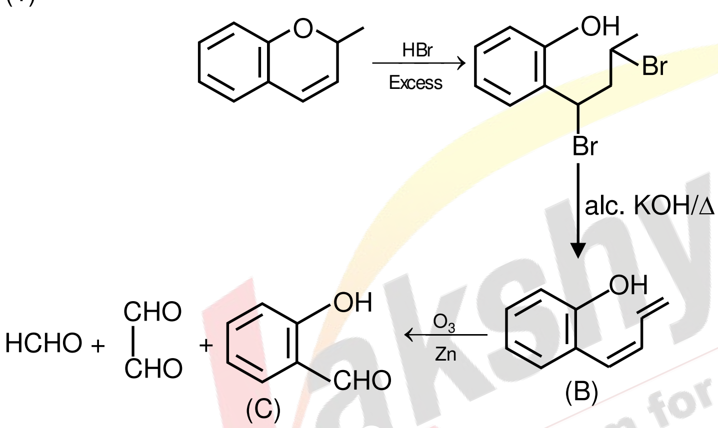
8. The major aromatic product C in the following reaction sequence will be :



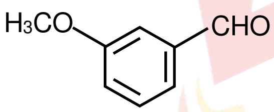
(1)  (2)  (3)  (4) 

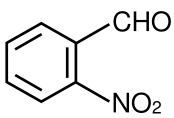
Ans. (1)

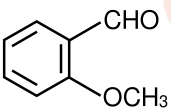
Sol.

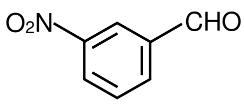


9. The increasing order of the following compounds towards HCN addition is :

(i) 

(ii) 

(iii) 

(iv) 

(1) (iii) < (i) < (iv) < (ii) (2) (i) < (iii) < (iv) < (ii) (3) (iii) < (iv) < (ii) < (i) (4) (iii) < (iv) < (i) < (ii)

Ans. **(1)**

Sol. –I, – M effect of NO_2 increase reactivity towards nucleophilic addition reaction with HCN.

10. In general, the property (magnitudes only) that shows an opposite trend in comparison to other properties across a period is :

(1) Electronegativity (2) Electron gain enthalpy

(3) Ionization enthalpy (4) Atomic radius

Ans. **(4)**

Sol. On moving Left to Right along a period.
Atomic Radius $\Rightarrow$ decreases.
Electronegativity $\Rightarrow$ Increases.
Electron gainenthalpy $\Rightarrow$ Increases.
Ionisation Enthalpy $\Rightarrow$ Increases.

11. For octahedral Mn(II) and tetrahedral Ni(II) complexes, consider the following statements :
(I) both the complexes can be high spin.
(II) Ni(II) complex can very rarely be of low spin.
(III) with strong field ligands, Mn(II) complexes can be low spin.
(IV) aqueous solution of Mn(II) ions is yellow in color.
The correct statements are :
(1) (I) and (II) only
(2) (I), (II) and (III) only
(3) (I), (III) and (IV) only
(4) (II), (III) and (IV) only

Ans. **(2)**

Sol. With weak field ligands Mn(II) will be of high spin and with strong field ligands it will be of low spin. Ni(II) tetrahedral complexes will be genrally of high spin due to sp3 hybridisation. Mn(II) is of light pink color in aqueous solution.

12. In Carius method of estimation of halogen, 0.172 g of an organic compound showed presence of 0.08 g of bromine. Which of these is the correct structure of the compound ?

(1) $H_3C–Br$
(2) $H_3C–CH_2–Br$
(3) 

(4) 

Ans. **(3)**

Sol. Mole of Bromine = $\frac{0.08}{80} = 10^{-3}$ mole

Molar mass of compound = $\frac{0.172}{M} = 10^{-3}$

$$M = \frac{0.172}{10^{-3}} = 172\text{gm}$$

Molar mass of 

= 80 + 72 + 6 + 14 = 172 gm

13. An open beaker of water in quilibrium with water vapour is in a sealed container. When a few grams of glucose are added to the beaker of water, the rate at which water molecules :
(1) leaves the vapour decreases
(2) leaves the solution decreases
(3) leaves the vapour increases
(4) leaves the solution increases

Ans. **(3)**

Sol. The vapour pressure of solution will be less than vapour pressure of pure solvent, so some vapour molecules will get condensed to maintain new equilibrium.

14. The statement that is not true about ozone is :

(1) in the stratosphere, CFCs release chlorine free radicals (Cl) which reacts with O_3 to give chlorine dioxide radicals.

(2) in the stratosphere, it forms a protective shield against UV radiation.

(3) it is a toxic gas and its reaction with NO gives NO_2

(4) in the atmosphere, it is depleted by CFCs.

Ans. **(1)**

Sol. In presence of sunlight CFC's molecule divides & release chlorine free radical, which react with ozone give chlorine monoxide radical ($ClO^{\bullet}$) and oxygen.

$$CF_2Cl_2(g) \xrightarrow{UV} \dot{C}l(g) + \dot{C}F_2Cl(g)$$

$$Cl^{\bullet}(g) + O_3(g) \longrightarrow ClO^{\bullet}(g) + O_2(g)$$

$$ClO^{\bullet}(g) + O(g) \longrightarrow Cl^{\bullet}(g) + O_2(g)$$

15. The metal mainly used in devising photoelectric cells is :

(1) Li (2) Rb (3) Cs (4) Na

Ans. **(3)**

Sol^n Cesium has lowest ionisation enthalpy and hence it can show photoelectric effect to the maximum extent hence it is used in photo electric cell.

16. Which of the following is used for the preparation of colloids ?

(1) Ostwald process (2) Van Arkel Method

(3) Mond Process (4) Bredig's Arc Method

Ans. **(4)**

Sol. Bredig's Arc Method is used for preparation of colloidal sol's of less reactive metal like Au, Ag, Pt.

17. Consider that d^6 metal ion (M^{2+}) forms a complex with aqua ligands, and the spin only magnetic moment of the complex is 4.90 BM. The geometry and the crystal field stabilization energy of the complex is :

(1) tetrahedral and $-1.6\Delta_t + 1P$ (2) octahedral and $-2.4\Delta_0 + 2P$

(3) tetrahedral and $-0.6\Delta_t$ (4) octahedral and $-1.6\Delta_0$

Ans. **(3)**

Sol. Since spin only magnetic moment is 4.90 BM so number of unpaired electrons must be 4. so If the complex is octahedral, then it has to be high spin complex with configuration $t_{2g}^{2,1,1}e_g^{1,1}$, in that case CFSE = 4 X $(-0.4\Delta_0)$ + 2 X 0.6 Δ_0 = -0.4 Δ_0

If the complex is tetrahedral then its electronic configuration will be = $e_g^{2,1}t_{2g}^{1,1,1}$ and CFSE will be = 3 X $(-0.6\ \Delta_t)$ + 3 X $(0.4\ \Delta_t)$ = $-0.6\Delta_t$

18. If AB_4 molecule is a polar molecule, a possible geometry of AB_4 is :

(1) Square pyramidal (2) Rectangular planar

(3) Square planar (4) Tetrahedral

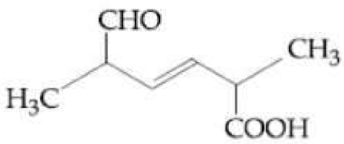
Ans. **(1)**

Sol^n. For AB_4 compound possible geometry are

S. No.	Bond pair	Lone pair	Total	Hybridisation	Geometry	Polarity
1	4	0	4	SP^3	Tetrahedral	non polar
2	4	1	5	SP^3d	Sea-saw	Polar
3	4	2	6	sp^3d^2	Square Planar	non polar

Square pyramidal can be polar due to lone pair moment as the bond pair moments will get cancelled out.

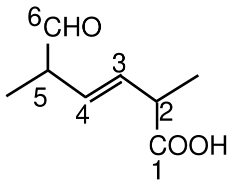
19. The IUPAC name for the following compound is :



(1) 2, 5-dimethyl-6-carboxy-hex-3-enal
(2) 2, 5-dimethyl-5-carboxy-hex-3-enal
(3) 6-formyl-2-methyl-hex-3-enoic
(4) 2, 5-dimethyl-6-oxo-hex-3-enoic acid

Ans. **(4)**

Sol.



(2, 5-dimethyl-6-oxo-hex-3-enoic acid

20. Consider the following reactions :

(i) Glucose + ROH $\xrightarrow{\text{dry HCl}}$ Acetal $\xrightarrow[(CH_3CO)_2O]{\text{x eq. of}}$ acetyl derivative

(ii) Glucose $\xrightarrow{Ni/H_2}$ A $\xrightarrow[(CH_3CO)_2O]{\text{y eq. of}}$ acetyl derivative

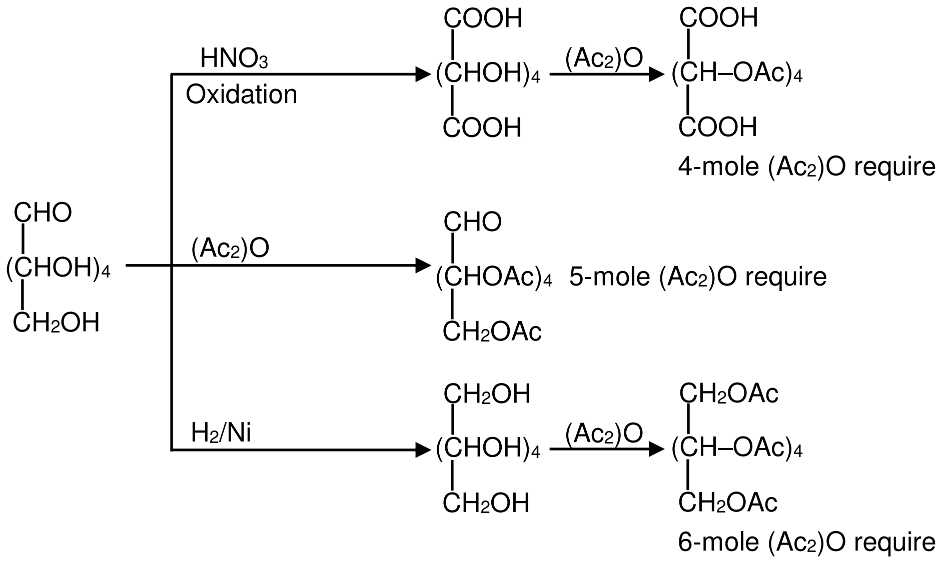
(iii) Glucose $\xrightarrow[(CH_3CO)_2O]{\text{z eq. of}}$ acetyl derivative

'x', 'y' and 'z' in these reactions are respectively.

(1) 5, 6 & 6
(2) 4, 5 & 5
(3) 4, 6 & 5
(4) 5, 4 & 5

Ans. **(3)**

Sol.



SECTION – 2 : (Maximum Marks : 20)

- This section contains **FIVE (05)** questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.
- If the numerical value has more than two decimal places **truncate/round-off** the value upto **TWO** decimal places.
  - Full Marks : **+4** If ONLY the correct option is chosen.
  - Zero Marks : **0** In all other cases

21. The mass of gas adsorbed, x, per unit mass of adsorbate, m, was measured at various pressures, p. A graph between log $\frac{x}{m}$ and log p gives a straight line with slope equal to 2 and the intercept equal to 0.4771. The value of $\frac{x}{m}$ at a pressure of 4 atm is :

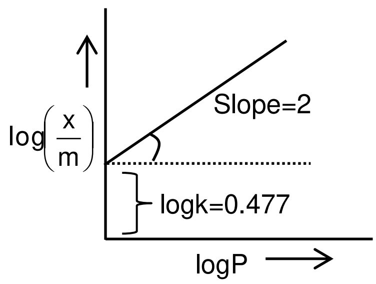
(Given log 3 = 0.4771)

Ans. **(48)**

Solⁿ.

$\left(\frac{x}{m}\right) = k(P)^{\frac{1}{n}}$

$\log\left(\frac{x}{m}\right) = \log k + \frac{1}{n}\log P$



Slope = $\frac{1}{n} = 2$ So $n = \frac{1}{2}$.

Intercept ⇒ logk = 0.477 So k = Antilog (0.477) = 3

So $\left(\frac{x}{m}\right) = k(P)^{\frac{1}{n}}$

$= 3[4]^2 = 48$

22. The oxidation states of iron atoms in compounds (A), (B) and (C), respectively, are x, y and z. The sum of x, y and z is ....

$\underset{(A)}{Na_4[Fe(CN)_5(NOS)]}$ $\underset{(B)}{Na_4[FeO_4]}$ $\underset{(C)}{[Fe_2(CO)_9]}$

Ans. (6)

Sol. The oxidation states of iron in these compounds will be

A = +2

B = +4

C = 0

The sum of oxidation states will be = 6.

23. The internal energy change (in J) when 90 g of water undergoes complete evaporation at 100ºC is...............

(Given : ΔH_{vap} for water at 373 K = 41 kJ/mol, R = 8.314 JK^{-1} mol^{-1})

Ans. **(189494)**

Sol. $\Delta H = \Delta U + \Delta n_g RT$

41000 X 5 = ΔU + 5 X 8.314 X 373

205000 = ΔU + 15505.61

ΔU = 189494.39 J = 189494 J

24. The Gibbs energy change (in J) for the given reaction at $[Cu^{2+}] = [Sn^{2+}] = 1$ M and 298 K is :

$Cu(s) + Sn^{2+}(aq.) \rightarrow Cu^{2+}(aq.) + Sn(s)$;

($E^0_{Sn^{2+}|Sn} = -0.16$ V, $E^0_{Cu^{2+}|Cu} = 0.34$ V, Take F = 96500 C mol^{-1})

Solⁿ **96500**

$E^0_{cell} = E^0_{Sn^{2+}/Sn} - E^0_{Cu^{2+}/Cu}$

$= -0.16 - 0.34$

$= -0.50V$

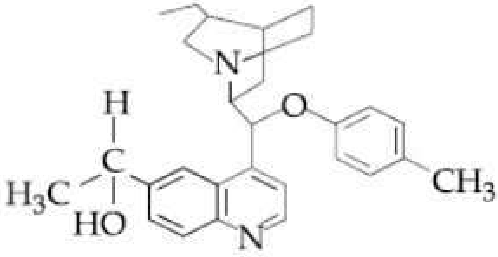
$\Delta G^0 = -nFE^0_{cell}$

$= -2 \times 96500 \times (-0.5)$

= 96500J

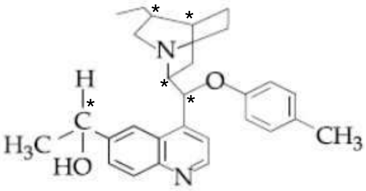
= 96.5 KJ = 96500 J

25. The number of chiral carbons present in the molecule given below is……………



Ans. (5)

Sol.



PART : MATHEMATICS

SECTION – 1 : (Maximum Marks : 80)

Straight Objective Type

This section contains **20 multiple choice questions.** Each question has 4 choices (1), (2), (3) and (4) for its answer, out of which **Only One** is correct.

1. The sum of the first three terms of a G.P. is S and their product is 27. Then all such S lie in

(1) $(-\infty, -3] \cup [9, \infty)$ (2) $(-\infty, -9] \cup [3, \infty)$

(3) $[-3, \infty)$ (4) $(-\infty, 9]$

Ans. (1)

Sol. Let terms are $\frac{a}{r}$, a, ar

then $a^3 = 27 \Rightarrow a = 3$

Now $\frac{3}{r} + 3 + 3r = S$

$3\left(\frac{1}{r}+r\right)+3 = S$

$r + \frac{1}{r} \geq 2$

$3\left(\frac{1}{r}+r\right)+3 \in (-\infty, -3] \cup [9, \infty)$

2. Box I contains 30 cards numbered I to 30 and Box II contains 20 cards numbered 31 to 50. A box is selected at random and a card is drawn from it. The number on the card is found to be a non-prime number. The probability that the card was drawn from Box I is :

(1) $\frac{2}{3}$ (2) $\frac{2}{5}$ (3) $\frac{8}{17}$ (4) $\frac{4}{17}$

Ans. (3)

Sol. $P(B_1) = \frac{1}{2} = P(B_2)$

P(Non-prime) $= P(B_1).\ P(N.P/B_1) + P(B_2).\ P(N.P/B_2)$

$= \frac{1}{2}.\frac{20}{30}+\frac{1}{2}.\frac{15}{20}$

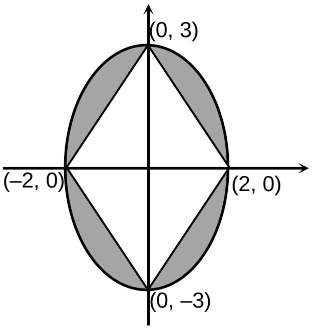
$P(B_1/N.P) = \dfrac{\frac{1}{2}.\frac{20}{30}}{\frac{1}{2}.\frac{20}{30}+\frac{1}{2}.\frac{15}{20}} = \frac{8}{17}$

3. Area (in sq. units) of the region outside $\frac{|x|}{2}+\frac{|y|}{3}=1$ and inside the ellipse $\frac{x^2}{4}+\frac{y^2}{9}=1$ is :

(1) $3(\pi - 2)$ (2) $3(4 - \pi)$ (3) $6(\pi - 2)$ (4) $6(4 - \pi)$

Ans. (3)

Sol.



Area of Ellipse = $\pi ab = 6\pi$

Area enclose by $\frac{|x|}{2}+\frac{|y|}{3}=1$ is

$= \frac{1}{2}(d_1 d_2) = \frac{1}{2}(4)(6) = 12$

so required Area is = $6\pi - 12$

4. The domain of the function $f(x) = \sin^{-1}\left(\frac{|x|+5}{x^2+1}\right)$ is $(-\infty, -a] \cup [a, \infty)$, Then a is equal to :

(1) $\frac{1+\sqrt{17}}{2}$ (2) $\frac{\sqrt{17}-1}{2}$ (3) $\frac{\sqrt{17}}{2}$ (4) $\frac{\sqrt{17}}{2}+1$

Ans. (1)

Sol. $\left|\frac{|x|+5}{x^2+1}\right| \le 1$

$|x| + 5 \le x^2 + 1$

$x^2 - |x| - 4 \ge 0$

Let $|x| = t \Rightarrow t^2 - t - 4 \ge 0$

$\left(|x|+\frac{\sqrt{17}-1}{2}\right)\left(|x|-\frac{\sqrt{17}+1}{2}\right) \ge 0$

$x \in \left(-\infty, -\frac{\sqrt{17}+1}{2}\right] \cup \left[\frac{\sqrt{17}+1}{2}, \infty\right)$

$a = \frac{1+\sqrt{17}}{2}$

5. If $|x| < 1$, $|y| < 1$ and $x \ne y$, then the sum to infinity of the following series $(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots$ is :

(1) $\frac{x+y+xy}{(1-x)(1-y)}$ (2) $\frac{x+y-xy}{(1-x)(1-y)}$ (3) $\frac{x+y-xy}{(1-x)(1+y)}$ (4) $\frac{x+y+xy}{(1+x)(1+y)}$

Ans. (2)

Sol. $(x + y) + (x^2 + xy + y^2) + (x^3 + x^2y + xy^2 + y^3) + \ldots$

$= \frac{1}{x-y}\left(\frac{x^2}{1-x}-\frac{y^2}{1-y}\right) = \frac{1}{x-y}\left(\frac{x^2-x^2y-y^2+xy^2}{(1-x)(1-y)}\right) = \frac{x+y-xy}{(1-x)(1-y)}$

6. The value of $\left(\dfrac{1+\sin\frac{2\pi}{9}+i\cos\frac{2\pi}{9}}{1+\sin\frac{2\pi}{9}-i\cos\frac{2\pi}{9}}\right)^3$ is :

(1) $-\frac{1}{2}(\sqrt{3}-i)$ (2) $\frac{1}{2}(\sqrt{3}-i)$ (3) $\frac{1}{2}(1-i\sqrt{3})$ (4) $-\frac{1}{2}(1-i\sqrt{3})$

Ans. (1)

Sol. $\left(\dfrac{1+\cos\frac{5\pi}{18}+i\sin\frac{5\pi}{18}}{1+\cos\frac{5\pi}{18}-i\sin\frac{5\pi}{18}}\right)^3 = \left(\dfrac{2\cos^2\frac{5\pi}{36}+i2\sin\frac{5\pi}{36}.\cos\frac{5\pi}{36}}{2\cos^2\frac{5\pi}{36}-i2\sin\frac{5\pi}{36}.\cos\frac{5\pi}{36}}\right)^3$

$= \left(\dfrac{\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}}{\cos\frac{5\pi}{36}-i\sin\frac{5\pi}{36}}\right)^3 = \left(\cos\frac{5\pi}{36}+i\sin\frac{5\pi}{36}\right)^6$

$= \cos\left(6\times\frac{5\pi}{36}\right)+i\sin\left(6\times\frac{5\pi}{36}\right) = \cos\frac{5\pi}{6} + i\sin\frac{5\pi}{6} \Rightarrow -\frac{\sqrt{3}}{2}+i\frac{1}{2}$

7. Let y = y(x) be the solution of the differential equation, $\frac{2+\sin x}{y+1}.\frac{dy}{dx} = -\cos x$, y > 0, y(0) = 1. If y(π) = a and $\frac{dy}{dx}$ at x = π is b, then the ordered pair (a, b) is equal to :

(1) $\left(2, \frac{3}{2}\right)$ (2) (1, –1) (3) (2, 1) (4) (1, 1)

Ans. (4)

Sol. $\frac{dy}{1+y} = \frac{-\cos x}{2+\sin x}dx$

$\ell n(1 + y) = -\ell n(2 + \sin x) + \ell nc$

$(1 + y)(2 + \sin x) = c$

$4 . 1 = c \Rightarrow c = 4$

$1 + y = \frac{4}{2+\sin x} \Rightarrow y = \frac{4}{2+\sin x} - 1$

$y(\pi) = 2 - 1 = 1 = a$

$\frac{dy}{dx} = \frac{-4}{(2+\sin x)^2}.\cos x = 1$ at $x = \pi \Rightarrow b = 1$

(a, b) = (1, 1)

8. The plane passing through the points (1, 2, 1), (2, 1, 2) and parallel to the line, 2x = 3y, z = 1 also passes through the point :

(1) (2, 0, –1) (2) (0, 6, –2) (3) (0, –6, 2) (4) (–2, 0, 1)

Ans. (4)

Sol. Plane passes through (2, 1, 2) is

$a(x - 2) + b(y - 1) + (z - 2) = 0$

it also passes through (1, 2, 1)

$\therefore$ $-a + b - c = 0 \Rightarrow a - b + c = 0$ ....(1)

given line

$\frac{x}{3} = \frac{y}{2} = \frac{z-1}{0}$ is parallel to (1)

$\therefore$ $3a + 2b + c0 = 0 \Rightarrow$

$$\frac{a}{0-2} = \frac{b}{3-0} = \frac{c}{2+3}$$

$$\frac{a}{2} = \frac{b}{-3} = \frac{c}{2+3}$$

$$\frac{a}{2} = \frac{b}{-3} = \frac{c}{-5}$$

$\therefore$ plane is $2x - 4 - 3y + 3 - 5z + 10 = 0 \Rightarrow 2x - 3y - 5z + 9 = 0$

9. Let $\alpha > 0, \beta > 0$ be such that $\alpha^3 + \beta^2 = 4$. If the maximum value of the term independent of x in the binomial expansion of $\left(\alpha x^{1/9} + \beta x^{-1/6}\right)^{10}$ is 10k, then k is equal to :

(1) 176 (2) 336 (3) 352 (4) 84

Ans. (2)

Sol. $T_{r+1} = {}^{10}C_r (\alpha x^{1/9})^{10-r} (\beta x^{-1/6})^r$

$T_{r+1} = {}^{10}C_r\, \alpha^{10-r} \beta^r (x)^{\frac{10-r}{9} - \frac{r}{6}}$

Term independent of x

$$\frac{10-r}{9} - \frac{r}{6} = 0 \Rightarrow r = 4$$

$T_5 = {}^{10}C_4\, \alpha^6 \beta^4$

Now Let α^3, β^2 are 2 numbers.

$A \geq G$

$\Rightarrow \frac{\alpha^3 + \beta^2}{2} \geq (\alpha^3 \beta^2)^{1/2}$

$\Rightarrow \alpha^3 \beta^2 \leq 4$

$\Rightarrow \alpha^6 \beta^4 \leq 16$

$\Rightarrow \frac{T_5}{{}^{10}C_4} \leq 6$

$\Rightarrow T_5 \leq 16 \,.\, {}^{10}C_4$

$\Rightarrow T_{5\,max} = 16 \times {}^{10}C_4 = 10K$

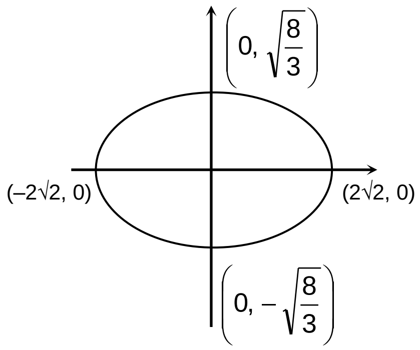
$\Rightarrow K = 336$

10. If $R = \{(x, y) : x, y \in Z, x^2 + 3y^2 \leq 8\}$ is a relation on the set of integers Z, then the domain R^{-1} is ;

(1) $\{-1, 0, 1\}$ (2) $\{-2, -1, 1, 2\}$ (3) $\{0, 1\}$ (4) $\{-2, -1, 0, 1, 2\}$

Ans. (1)

Sol. $\frac{x^2}{8} + \frac{y^2}{8/3} \le 1$



Domain of $R^{-1} \equiv$ Range of R $\equiv$ Value of y $\equiv \{-1, 0, 1\}$

11. If p(x) be a polynomial of degree three that has a local maximum value 8 at x = 1 and a local minimum value 4 at x = 2; then p(0) is equal to

(1) –24 (2) –12 (3) 6 (4) 12

Ans. (2)

Sol. Clearly $P'(x) = \lambda\,(x-1)(x-2)$ where $\lambda > 0$

$$P(x) = \lambda\left[\frac{x^3}{3} - \frac{3x^2}{2} + 2x\right] + C$$

given $P(1) = 8 \Rightarrow \lambda\left(\frac{1}{3} - \frac{3}{2} + 2\right) + C = 8 \qquad \Rightarrow \frac{5\lambda}{6} + C = 8$ ....(i)

also $P(2) = 4 \Rightarrow \lambda\left(\frac{8}{3} - 6 + 4\right) + C = 4 \qquad \Rightarrow \frac{2}{3}\lambda + C = 4$ ....(ii)

By (i) and (ii) $\Rightarrow C = -12$

$\Rightarrow P(0) = -12$

12. A line parallel to the straight line $2x - y = 0$ is tangent to the hyperbola $\frac{x^2}{4} - \frac{y^2}{2} = 1$ at the point (x_1, y_1).

Then $x_1^2 + 5y_1^2$ is equal to :

(1) 10 (2) 5 (3) 8 (4) 6

Ans. (4)

Sol. Tangent at (x_1, y_1)

$xx_1 - 2yy_1 - 4 = 0$

This is parallel to $2x - y = 0$

$\Rightarrow \frac{x_1}{2y_1} = 2$

$\Rightarrow x_1 = 4y_1$ ....(1)

Point (x_1, y_1) lie on hyperbola.

$\frac{x_1^2}{4} - \frac{x_1^2}{2} - 1 = 0$ ....(2)

On solving eq. (1) and (2)

We get $x_1^2 + 5y_1^2 = 6$

13. The contrapositive of the statement "If I reach the station in time, then I will catch the train" is:
(1) If ¡ will catch the train, then I reach the station in time.
(2) If do not reach the station in time, then ¡ will not catch the train.
(3) If I do not reach the station in time, then I will catch the train.
(4) If I will not catch the train, then I do not reach the station in time.

Ans. (4)

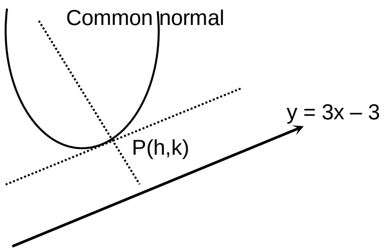
Sol. Contrapositive of $p \to q$ is $\sim q \to \sim p$

14. Let P(h, k) be a point on the curve $y = x^2 + 7x + 2$, nearest to the line, $y = 3x - 3$. Then the equation of the normal to the curve at P is :

(1) $x + 3y - 62 = 0$ (2) $x + 3y + 26 = 0$ (3) $x - 3y - 11 = 0$ (4) $x - 3y + 22 = 0$

Ans. (2)

Sol.



Tangent at p(h, k) will be parallel to given line $\left.\frac{dy}{dx}\right|_{(h,k)} = 2h + 7 = 3 \Rightarrow h = -2$

Point P lies on curve $K = (-2)^2 - 7 \times 2 + 2 = -8$

Normal at P(–2, –8), normal slop $= -\frac{1}{3}$

$x + 3y + 26 = 0$

15. Let A be a 2 × 2 real matrix with entries from {0, 1} and $|A| \neq 0$. Consider the following two statements ;

(P) If $A \neq I_2$, then $|A| = -1$

(Q) If $|A| = 1$, then $tr(A) = 2$

where I_2 denotes 2 × 2 identity matrix and tr(A) denotes the sum of the diagonal entries of A. Then :

(1) (P) is true and (Q) are false (2) Both (P) and (Q) are true
(3) Both (P) and (Q) are false (4) (P) is false and (Q) is true

Ans. (2)

Sol. Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ $a, b, c, d \in \{0, 1\}$

$|A| = ad - bc \neq 0$

$\Rightarrow$ $ad = 1, bc = 0$ or $ad = 0, bc = 1$

(P) If $A \neq I_2 \Rightarrow$ $ad \neq 1 \Rightarrow$ $ad = 0, bc = 1 \Rightarrow$ $|A| = -1$ (P) is true.

(Q) If $A = I \Rightarrow$ $ad = 1 \Rightarrow$ $ad = 1, bc = 0 \Rightarrow$ $tr(A) = 2$ (Q) is true.

16. Let S be the set of all $\lambda \in R$ for which the system of linear equations

$2x - y + 2z = 2$

$x - 2y + \lambda z = -4$

$x + \lambda y + z = 4$

has no solution. then the set S

(1) is a singleton
(2) contains exactly two elements
(3) contains more than two elements
(4) is an empty set

Ans. (2)

Sol. For no. solution $\Delta = 0$ and at least one of $\Delta_1, \Delta_2, \Delta_3$ is non-zero.

$$\Delta = \begin{vmatrix} 2 & -1 & 2 \\ 1 & -2 & \lambda \\ 1 & \lambda & 1 \end{vmatrix} = -(\lambda - 1)(2\lambda + 1)$$

$$\Delta_1 = \begin{vmatrix} 2 & -1 & 2 \\ -4 & -2 & \lambda \\ 4 & \lambda & 1 \end{vmatrix} = -2(\lambda^2 + 6\lambda - 4)$$

$$\Delta = 0 \quad \Rightarrow \quad \lambda = 1, -\frac{1}{2}$$

Hence, $S = \{1, -\frac{1}{2}\}$

17. If the tangent to the curve $y = x + \sin y$ at a point (a, b) is parallel to the line joining $\left(0, \frac{3}{2}\right)$ and $\left(\frac{1}{2}, 2\right)$, then

(1) $|b - a| = 1$
(2) $b = \frac{\pi}{2} + a$
(3) $|a + b| = 1$
(4) $b = a$

Ans. (1)

Sol. $y = x + \sin y$

$$\frac{dy}{dx} = \frac{1}{1 - \cos y} = \frac{\frac{1}{2} - 0}{2 - \frac{3}{2}} = 1$$

$$\Rightarrow \quad \cos y = 0 \quad \Rightarrow \quad y = (2n + 1)\frac{\pi}{2}$$

Point lie on curve $b = a + \sin y$

$b - a = \sin y$

$|b - a| = 1$

18. If a function f(x) defined by

$$f(x) = \begin{cases} ae^x + be^{-x} & , & -1 \le x < 1 \\ cx^2 & , & 1 \le x \le 3 \\ ax^2 + 2cx & , & 3 < x \le 4 \end{cases}$$

be continuous for some $a, b, c \in R$ and $f'(0) + f'(2) = e$, then the value of a is :

(1) $\frac{1}{e^2 - 3e + 13}$
(2) $\frac{e}{e^2 - 3e + 13}$
(3) $\frac{e}{e^2 - 3e - 13}$
(4) $\frac{e}{e^2 + 3e + 13}$

Ans. (2)

Sol. Continuous at x = 1, 3

$f(1) = f(1^+) \quad \Rightarrow \quad ae + be^{-1} = c$ .....(1)

$f(3) = f(3^+) \quad \Rightarrow \quad 9c = 9a + 6c \quad \Rightarrow \quad c = 3a$ .....(2)

From (1) and (2)

$b = ae(3 - e)$ .....(3)

$$f'(x) = \begin{cases} ae^x - be^{-x} & -1 < x < 1 \\ 2cx & 1 < x < 3 \\ 2ax + 2c & 3 < x < 4 \end{cases}$$

$f'(0) = a - b,\ f'(2) = 4c$

Given $f'(0) + f'(2) = e$

$a - b + 4c = e$ .....(4)

by using eq. (1), (2), (3) & (4)

$$a = \frac{e}{e^2 - 3e + 13}$$

19. Let α and β be the roots of the equation, $5x^2 + 6x - 2 = 0$. If $S_n = \alpha^n + \beta^n$, n = 1, 2, 3,....., then :

(1) $6S_6 + 5S_5 = 2S_4$

(2) $5S_6 + 6S_5 = 2S_4$

(3) $5S_6 + 6S_5 + 2S_4 = 0$

(4) $6S_6 + 5S_5 + 2S_4 = 0$

Ans. (2)

Sol. $5\alpha^2 + 6\alpha = 2$

$5S_6 + 6S_5 = \alpha^4(5\alpha^2 + 6\alpha) + \beta^4(5\beta^2 + 6\beta)$

$= 2(\alpha^4 + \beta^4) = 2S_4$

20. Let X = {x ∈ N : 1 ≤ 1 ≤ 17} and Y = {ax + b : x ∈ X and a, b ∈ R, a > 0}. If mean and variance of elements of Y are 17 and 216 respectively then a + b is equal to :

(1) 7

(2) 9

(3) –7

(4) –27

Ans. (3)

Sol. $B(\bar{x}) = a\bar{x} + b = \frac{a(1+2+3+....+17)}{17} + b = 17$

$\frac{a.(17.18)}{17.2} + b = 17$

$9a + b = 17$ ....(i)

$\sigma A^2 = \frac{\Sigma x^2}{n} - \left(\frac{\Sigma x}{n}\right)^2 = \frac{1^2+2^2+....+17^2}{17} - \left(\frac{1+2+....+17}{17}\right)^2$

$= \frac{17.18.35}{6.17} - \left(\frac{17.18}{2.17}\right)^2$

$= 105 - 81 = 24$

$\therefore \sigma^2_B = a^2\, \sigma_A^2 = a^2.24 = 216$

$a^2 = \frac{216}{24} = 9$

$a = 3 \ \therefore\ b = 17 - 27$

$\therefore\ b = 17 - 27$

$b = -10$

$\therefore\ a + b = -7$

SECTION – 2 : (Maximum Marks : 20)

- This section contains **FIVE (05)** questions. The answer to each question is **NUMERICAL VALUE** with two digit integer and decimal upto one digit.
- If the numerical value has more than two decimal places **truncate/round-off** the value upto **TWO** decimal places.
  - Full Marks : **+4** If ONLY the correct option is chosen.
  - Zero Marks : **0** In all other cases

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21. Let $\vec a, \vec b$ and $\vec c$ be three unit vectors such that $|\vec a-\vec c|^2+|\vec a-\vec c|^2=8$. Then $|\vec a+2\vec b|^2+|\vec a+2\vec c|^2$ is equal to

Ans. (02.00)

Sol. $|\vec a|=|\vec b|=|\vec c|=1$

$|\vec a-\vec b|^2+|\vec a-\vec c|^2=8$

$\Rightarrow \quad \vec a.\vec b+\vec a.\vec c=-2$

Now $\quad |\vec a+2\vec b|^2+|\vec a+2\vec c|^2 = 2|\vec a|^2 + 4|\vec b|^2 + 4|\vec c|^2 + 4(\vec a.\vec b+\vec a.\vec c) = 2$

22. If $\lim\limits_{x\to1}\dfrac{x+x^2+x^3+...+x^n-n}{x-1} = 820$, $(n \in N)$ then the value of n is equal to …….

Ans. (40.00)

Sol. $\lim\limits_{x\to1}\dfrac{x+x^2+x^3+....+x^n-n}{x-1}=820\left(\dfrac{0}{0}\right)$

$\lim\limits_{x\to1}\dfrac{1+2x+3x^2+.....+nx^{n-1}}{1}=820$

$\Rightarrow 1 + 2 + 3 + ... + n = 820$

$\Rightarrow \dfrac{n(n+1)}{2}=820$

$\Rightarrow n^2 + n – 1640$

$\Rightarrow n^2 + n – 1640 = 0$

$\Rightarrow n = 40 \qquad n \in N$

23. The number of integral values of k for which the line, $3x + 4y = k$ intersects the circle, $x^2 + y^2 - 2x - 4y + 4 = 0$ at two distinct points is ….

Ans. (09.00)

Sol. If line cuts circle then $p < r$

Centre of circle (1, 2), $r = 1$

$\left|\frac{3+8-k}{5}\right| < 1 \quad \Rightarrow \quad k \in (6, 16)$

$k = 7, 8, 9, 10, 11, 12, 13, 14, 15, \ldots\ldots\ldots$

24. If the letters of the word 'MOTHER' be permuted and all the words so formed (with or without meaning) be listed as in a dictionary, then the position of the word 'MOTHER' is ……

Ans. (309.00)

Sol. M O T H E R

3 4 6 2 1 5 $\Rightarrow$ 2 5! + 2 4! + 3 3! + 2! + 1 = 240 + 48 + 18 + 2 + 1 = 309

25. The integral $\int_0^2 ||x-1|-x|dx$ is equal to ;

Ans. (01.50)

Sol. $\int_0^2 ||x-1|-x|dx = \int_0^1 |1-x-x|dx + \int_1^2 |x-1-x|dx$

$= \int_0^{1/2}(1-2x)dx + \int_{1/2}^1 (2x-1)dx + \int_1^2 dx$

$= \left[x-x^2\right]_0^{\frac{1}{2}} + \left[x^2-x\right]_{\frac{1}{2}}^1 + [x]_1^2 = \frac{1}{2} - \frac{1}{4} + (1-1) - \left(\frac{1}{4} - \frac{1}{2}\right) + 2 - 1 = \frac{1}{4} + \frac{1}{4} + 1 = \frac{3}{2}$